

ASSIGNMENT ON DISCRETE MATHEMATICS

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Graph theory

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Abstract

In this article, we will discuss about graph colouring algorithm. Graph theory has many applications in day-to-day life, in which scheduling problem. Is most common problem that can be solved using colouring concept. In this project we are going to try to get solution of real-life problems like scheduling using graph theory. We will show you analytical as well as mathematical solution for this problem. We will show you other application of this project. Then we will conclude our project and we will also share references we use to make this awesome project.

1 INTRODUCTION

What do you think if there is only one subject to learn? what do you think if there is no any kind of standard class in any institute?

You would answer that, if it is so, there will be no need for time management or any kind of timetable.

But we know that, it's never possible for the developed or developing nation where school, colleges are restricted only for one subject to teach.

So, for multiple subjects as well as for multiple standards in school, college, there should be proper time-table to manage every class.

For managing those time-table. Nowadays, some experts use graph theory as a practical tool to scheduling time-table for any event.

Graph theory is very well-known topic of Discrete mathematics and graph colouring (Vertex colouring) is very well-known topic of graph theory.

We will discuss about it and solve it by graph theory and particularly by colouring algorithm or method. In colouring we use Vertex colouring method for scheduling.

2 A FORMAL STATEMENT OF THE PROBLEM

here we have a problem for scheduling times for lectures in a university. University has nine different subjects. They have a list of students who has applied for different subject.

there are many lectures with common student. You want to make sure that any two lectures with a common student occur at different times to avoid a conflict.

- **Here is the list of subjects:**

Astronomy, Chemistry, Greek, History, Italian, Latin, Music, Philosophy, Spanish

- **We also have a list of lectures with the common subject:**

Astronomy: Chemistry, Greek, History and Music

Chemistry: Astronomy and Spanish <https://www.overleaf.com/project/60dc8fc74>

Greek: Astronomy, History, Latin, Music, Philosophy

History: Astronomy, Greek, Latin, Spanish

Italian: Latin, Music, Spanish

Latin: Greek, History, Italian, Music, Philosophy, Spanish

Music: Astronomy, Greek, Italian, Latin

Philosophy: Greek, Latin

Spanish: Chemistry, History, Italian, Latin

Now, we have to find proper time slots for these lectures.

3 MATHEMATICAL SOLUTION

For solution we have to describe the information such that we can easily convert it into graph. So, we had made a matrix of information so, we can easily convert it into graph.

Lecture	A	C	G	H	I	L	M	P	S
Astronomy	0	1	1	1	0	0	1	0	0
Chemistry	1	0	0	0	0	0	0	0	1
Greek	1	0	0	1	0	1	1	1	0
History	1	0	1	0	0	1	0	0	1
Italian	0	0	0	0	0	1	1	0	1
Latin	0	0	1	1	1	0	1	1	1
Music	1	0	1	0	1	1	0	0	0
Philosophy	0	0	1	0	0	1	0	0	0
Spanish	0	1	0	1	1	1	0	0	0

Here we have described matrix by 0 and 1 notation. In which, 1 represents that there are some students who have selected subjects with respective to column and row.

Now we will draw a graph according to this (9 X 9) matrix, where each vertex of the graph will describe the particular lectures. And connected edges between any two vertices will describe that there is a common student between them.

I. First, we will take all the lectures as vertex.

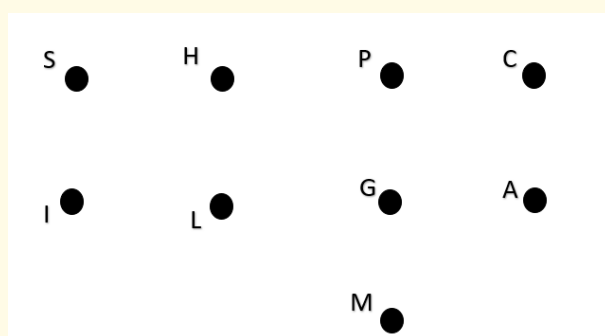


Figure 1: All subject as vertex

II. Now we will take one-by-one lectures and draw edges between the lectures.

- First, we take “astronomy”. Astronomy has common student with “chemistry”, “Greek”, “Music” and “History”.
- So, graph would look like:

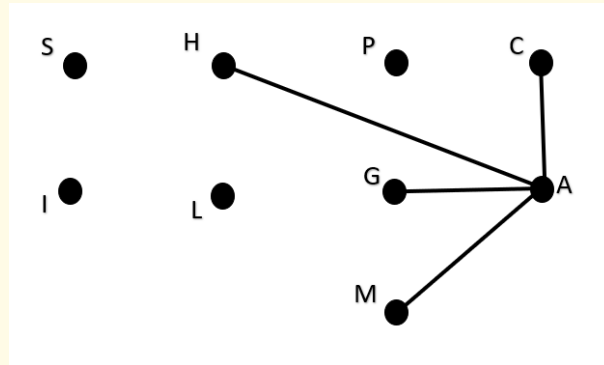


Figure 2: Adjacent vertex of "A"

- Now, we will take “Latin”. Latin has common student with “Spanish”, “Greek”, “Music”, “Italian”, “Philosophy” and “History”.
- So now, graph looks like:

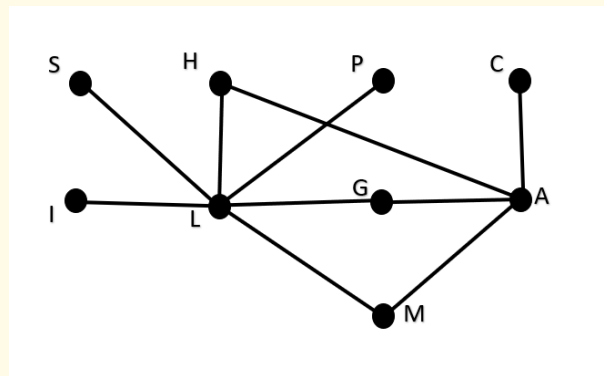


Figure 3: Adjacent vertex of "A" and "L"

- We will continue above process, until all the vertices connect to each other according to our matrix.

- After all, the graph looks like:

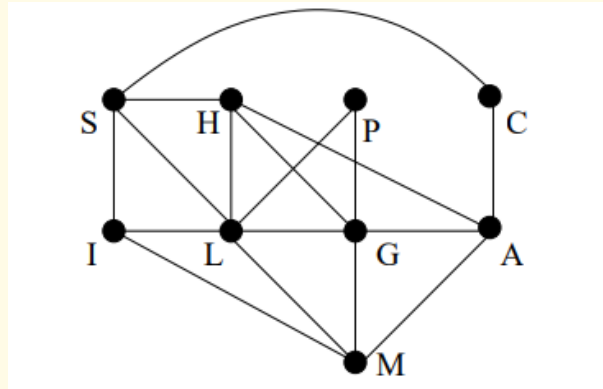


Figure 4: Full graph

- Now this is our final graph.
- In this graph we can see those adjacent vertices connected to each other have students in common.
- Now we need proper time slots for every lecture such that no students learning with multiple subjects miss any class. In short, we need to manage time-table.
- for that we will use graph colouring method.
- In this method we have to colour all the vertices such that no adjacent vertices get same colour.
- And our main target is that, we have to use as minimum colour as possible.
- Let take a vertex “A”. we are going to colour it with red.
- Now vertex “A” has adjacent vertex “G”, “C”, “M” and “H”. we can’t assign red colour to these four vertices.

- Now graph looks like:

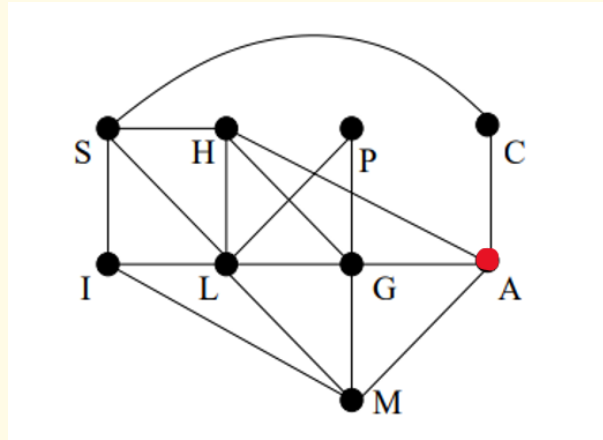


Figure 5: vertex "A" coloured graph

- But we can assign red colour to some of "L", "I", "S" and "P".
- Let assign red colour to vertex "L". now "I", "S" and "P" are its adjacent vertex. So, we can't assign red to it. So, we can assign red colour to vertex "A" and "L" only.
- Now graph looks like:

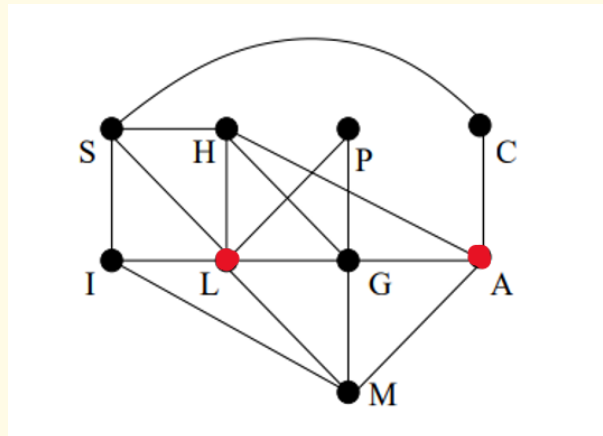


Figure 6: vertex "A" and "L" coloured graph

- Now we will assign blue colour to vertex "H". So, we can't assign blue colour to "S" and "G".
- But we can also assign blue colour to "P". because it is not the

adjacent vertex of “H”.

- Now “P” has only one adjacent vertex. Which is already coloured red.

- “I”, “C” and “M” are not adjacent to “H” and “P” .so we can assign blue to one of them.

- So, we assign blue colour to “M”.

- Now graph looks like:

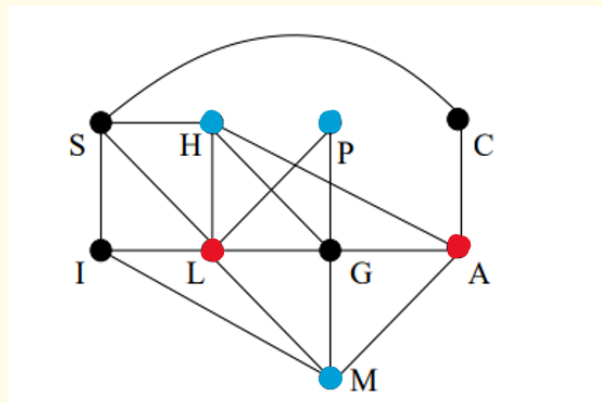


Figure 7: Red and Blue coloured graph

- Now we will colour all the vertex using above method.

- After all graph will looks like:

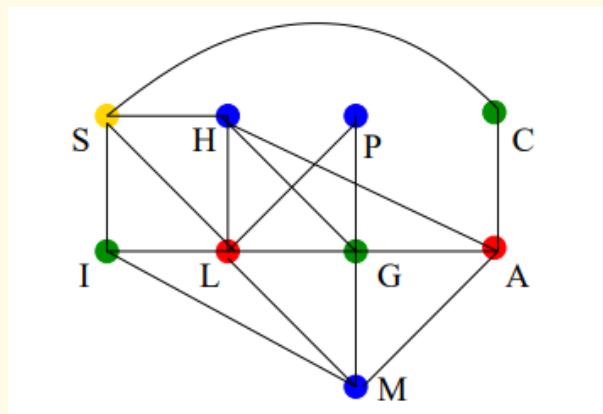


Figure 8: Coloured graph

- Remember, these colours denoting the time duration of that par-

ticular lecture assigned in vertex.

- Now here we can see that we have used 4 different colours to colour every vertex such that no vertices with same colour are connected to each other.

- So, it is found that there is four time-slots are possible for preventing conflict condition.

- We can assign 4 time-slots in a day with some duration of lecture time.

- During each time-slot, there may be more than one lectures are going on different classroom,

- But it is ensuring that, no students with multiple subjects would miss any lecture they taken.

- Hence problem solved.

4 APPLICATIONS OF THE PROBLEM:

- As you are familiar with the problem we had, this kind of problems are general in every educational institute. Using concept of graph theory and colouring method, usually institutes manage their academic time-table

- **Communication system:**

This concept limited not only to educational institutional sector, but nowadays it's become much more interesting tool to solve some other real-life problem like in communication system, suppose we want to minimize the number of radio frequencies we use while not using the same frequencies in nearby regions (to prevent interference). Then colours could represent frequencies, vertices could represent regions, and edges connect the vertices representing neighboring regions. Then we want to assign frequencies (i.e., colour the vertices) with no conflicts (i.e., no adjacent vertices have the same colour) using as few frequencies (i.e., colours) as possible.

- **Sudoku solving:**

This concept is also useful to solve some puzzles like Sudoku which is a logic-based puzzle.

- Other application like *sports scheduling, pattern matching, designing seating plans in various travelling service like airlines, train, bus, exam timetabling, the scheduling time-table for travelling system like railways, buses, taxis, etc.*

5 REFERENCES:

[1] Youtube Link for vertex coloring concept.

<https://youtu.be/kKtSJG0Kr4A>.

[2] Wikipedia Page for graph coloring.

https://en.wikipedia.org/wiki/Graph_coloring

[3] PDF Note.

http://web.math.princeton.edu/math_live/5/Notes2.pdf